Functions and Graphs - Questions

June 2017 Mathematics Advanced Paper 1: Pure Mathematics 3

1.

3.

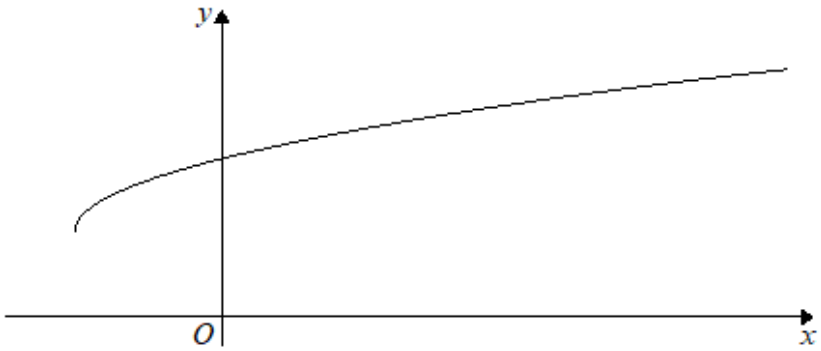

Figure 1

Figure 1 shows a sketch of part of the graph of $y = g(x)$, where

$$g(x) = 3 + \sqrt{x+2}\,, \quad x \geqslant -2$$

(*a*) State the range of g.

(1)

(*b*) Find $g^{-1}(x)$ and state its domain.

(3)

(*c*) Find the exact value of x for which

$$g(x) = x$$

(4)

(*d*) Hence state the value of a for which

$$g(a) = g^{-1}(a)$$

(1)

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2.

1. The functions f and g are defined by

$$f : x \to 7x - 1, \qquad x \in \mathbb{R},$$

$$g : x \to \frac{4}{x-2}, \qquad x \neq 2, x \in \mathbb{R},$$

(*a*) Solve the equation $fg(x) = x$.

(4)

(*b*) Hence, or otherwise, find the largest value of a such that $g(a) = f^{-1}(a)$.

(1)

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3.

2. Given that

$$f(x) = 2e^x - 5, \qquad x \in \mathbb{R},$$

(*a*) sketch, on separate diagrams, the curve with equation

(i) $y = f(x)$,

(ii) $y = |f(x)|$.

On each diagram, show the coordinates of each point at which the curve meets or cuts the axes.

On each diagram state the equation of the asymptote.

(6)

(*b*) Deduce the set of values of x for which $f(x) = |f(x)|$.

(1)

(*c*) Find the exact solutions of the equation $|f(x)| = 2$.

(3)

4.

7.

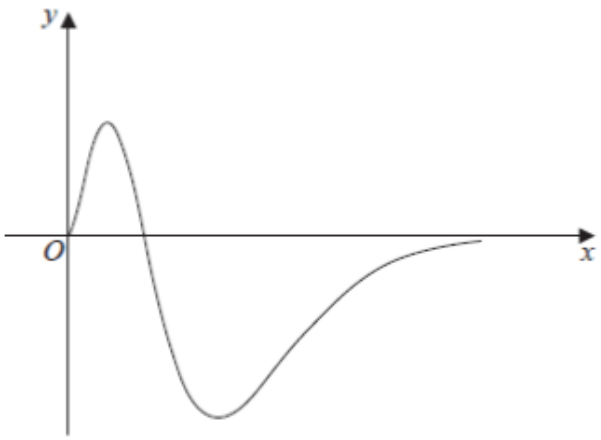

Figure 2

Figure 2 shows a sketch of part of the curve with equation

$$g(x) = x^2(1 - x)e^{-2x}, \qquad x \geq 0.$$

(*a*) Show that $g'(x) = f(x)e^{-2x}$, where $f(x)$ is a cubic function to be found.

(3)

(*b*) Hence find the range of g.

(6)

(*c*) State a reason why the function $g^{-1}(x)$ does not exist.

(1)

5.

4.

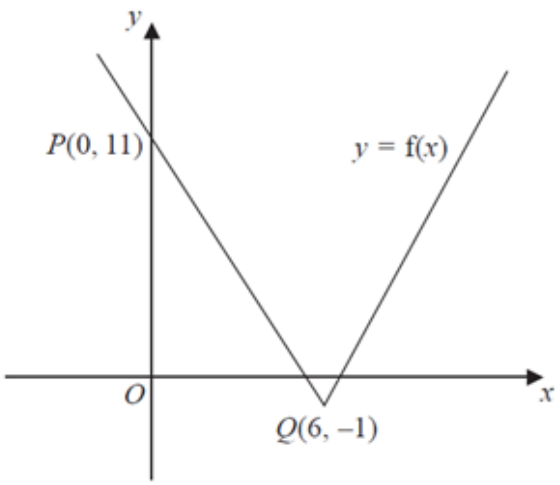

Figure 1

Figure 1 shows part of the graph with equation $y = f(x)$, $x \in \square$.

The graph consists of two line segments that meet at the point $Q(6, -1)$.

The graph crosses the y-axis at the point $P(0, 11)$.

Sketch, on separate diagrams, the graphs of

(*a*) $y = |f(x)|$

(2)

(*b*) $y = 2f(-x) + 3$

(3)

On each diagram, show the coordinates of the points corresponding to P and Q.

Given that $f(x) = a|x - b| - 1$, where a and b are constants,

(*c*) state the value of a and the value of b.

(2)

6.

5. $$g(x) = \frac{x}{x+3} + \frac{3(2x+1)}{x^2+x-6}, \qquad x > 3$$

(*a*) Show that $g(x) = \frac{x+1}{x-2}$, $x > 3$

(4)

(*b*) Find the range of g.

(2)

(*c*) Find the exact value of a for which $g(a) = g^{-1}(a)$.

(4)

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7.

2. Given that

$$f(x) = \ln x, \qquad x > 0$$

sketch on separate axes the graphs of

(i) $y = f(x)$,

(ii) $y = |f(x)|$,

(iii) $y = -f(x - 4)$.

Show, on each diagram, the point where the graph meets or crosses the x-axis. In each case, state the equation of the asymptote.

(7)

8.

7. The function f has domain $-2 \leq x \leq 6$ and is linear from $(-2, 10)$ to $(2, 0)$ and from $(2, 0)$ to $(6, 4)$. A sketch of the graph of $y = f(x)$ is shown in Figure 1.

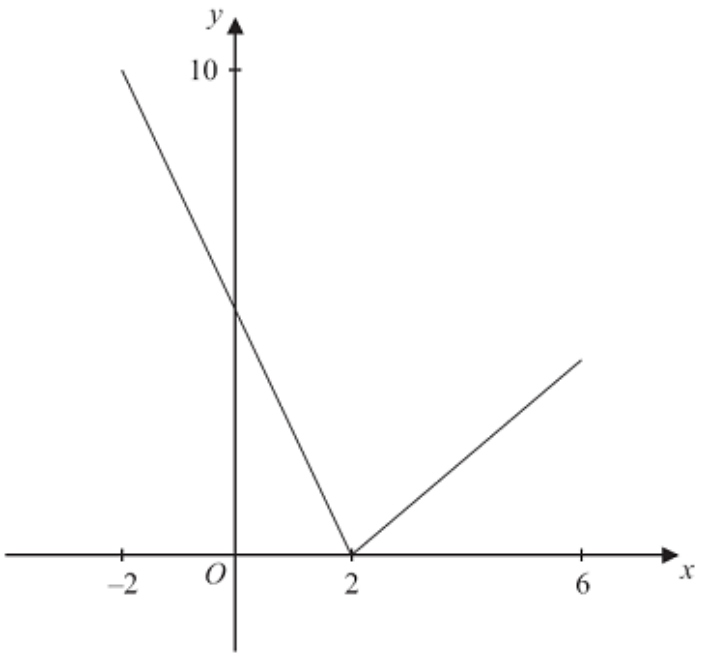

Figure 1

(*a*) Write down the range of f.

(1)

(*b*) Find ff(0).

(2)

The function g is defined by

$$g : x \to \frac{4+3x}{5-x}, \qquad x \in \mathbb{R}, \qquad x \neq 5.$$

(*c*) Find $g^{-1}(x)$.

(3)

(*d*) Solve the equation $gf(x) = 16$.

(5)

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9.

3.

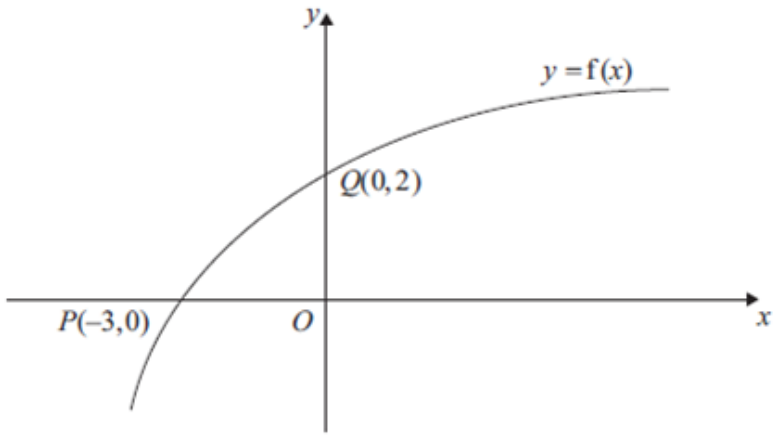

Figure 1

Figure 1 shows part of the curve with equation $y = \mathrm{f}(x)$, $x \in \mathbb{R}$.

The curve passes through the points $Q(0, 2)$ and $P(-3, 0)$ as shown.

(*a*) Find the value of $\mathrm{ff}(-3)$.

(2)

On separate diagrams, sketch the curve with equation

(*b*) $y = \mathrm{f}^{-1}(x)$,

(2)

(*c*) $y = \mathrm{f}(|x|) - 2$,

(2)

(*d*) $y = 2\mathrm{f}\left(\frac{1}{2}x\right)$.

(3)

Indicate clearly on each sketch the coordinates of the points at which the curve crosses or meets the axes.

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10.

4.

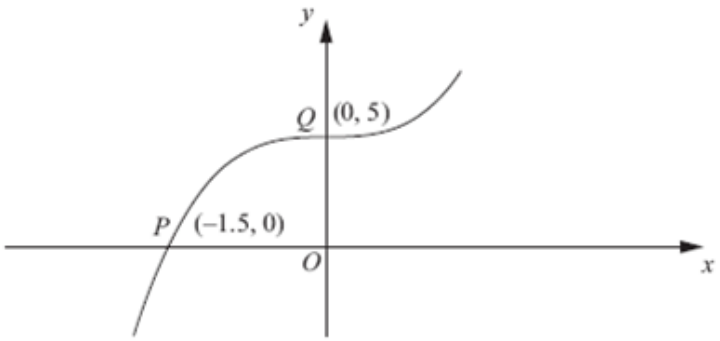

Figure 2

Figure 2 shows part of the curve with equation $y = f(x)$.
The curve passes through the points $P(-1.5, 0)$and $Q(0, 5)$ as shown.

On separate diagrams, sketch the curve with equation

(*a*) $y = |f(x)|$

(2)

(*b*) $y = f(|x|)$

(2)

(*c*) $y = 2f(3x)$

(3)

Indicate clearly on each sketch the coordinates of the points at which the curve crosses or meets the axes.

11.

6. The functions f and g are defined by

$$f: x \mapsto e^x + 2, \qquad x \in \mathbb{R},$$

$$g: x \mapsto \ln x, \qquad x > 0.$$

(*a*) State the range of f.

(1)

(*b*) Find fg(x), giving your answer in its simplest form.

(2)

(*c*) Find the exact value of x for which $f(2x + 3) = 6$.

(4)

(*d*) Find f^{-1}, the inverse function of f, stating its domain.

(3)

(*e*) On the same axes sketch the curves with equation $y = f(x)$ and $y = f^{-1}(x)$, giving the coordinates of all the points where the curves cross the axes.

(4)

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12.

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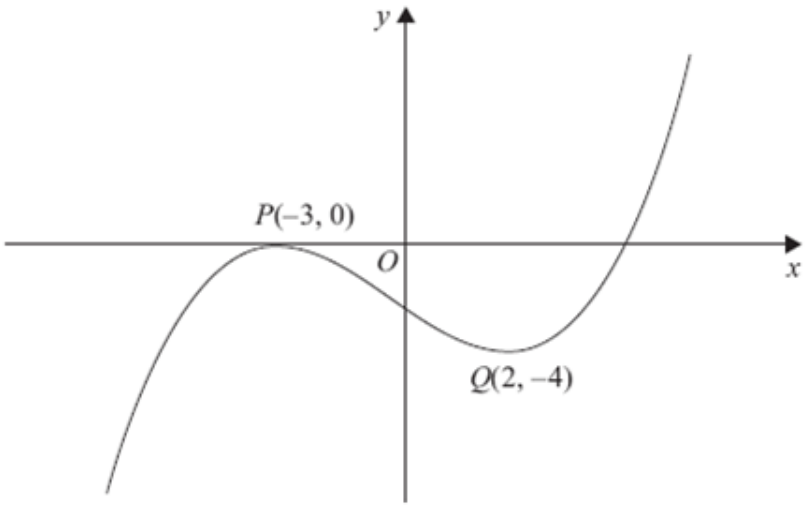

Figure 1

Figure 1 shows the graph of equation $y = f(x)$.

The points $P(-3, 0)$ and $Q(2, -4)$ are stationary points on the graph.

Sketch, on separate diagrams, the graphs of

(*a*) $y = 3f(x + 2)$,

(3)

(*b*) $y = |f(x)|$.

(3)

On each diagram, show the coordinates of any stationary points.

13.

7. The function f is defined by

$$f : x \mapsto \frac{3(x+1)}{2x^2 + 7x - 4} - \frac{1}{x+4}, \quad x \in \mathbb{R}, \ x > \frac{1}{2}.$$

(*a*) Show that $f(x) = \frac{1}{2x-1}$.

(4)

(*b*) Find $f^{-1}(x)$.

(3)

(*c*) Find the domain of f^{-1}.

(1)

$$g(x) = \ln (x + 1).$$

(*d*) Find the solution of $fg(x) = \frac{1}{7}$, giving your answer in terms of e.

(4)

14.

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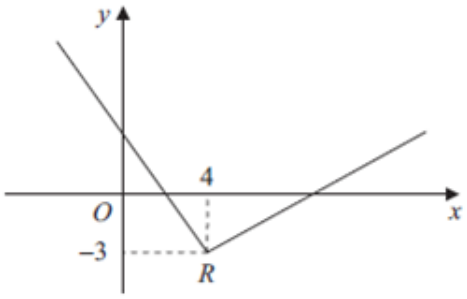

Figure 1

Figure 1 shows part of the graph of $y = f(x),\ x \in \mathbb{R}$.

The graph consists of two line segments that meet at the point $R\ (4, -3)$, as shown in Figure 1.

Sketch, on separate diagrams, the graphs of

(*a*) $y = 2f(x + 4)$,

(3)

(*b*) $y = |f(-x)|$.

(3)

On each diagram, show the coordinates of the point corresponding to R.

15.

4. The function f is defined by

$$f: x \mapsto 4 - \ln(x + 2), \quad x \in \mathbb{R}, \quad x \geq -1.$$

(*a*) Find $f^{-1}(x)$.

(3)

(*b*) Find the domain of f^{-1}.

(1)

The function g is defined by

$$g: x \mapsto e^{x^2} - 2, \quad x \in \mathbb{R}.$$

(*c*) Find $fg(x)$, giving your answer in its simplest form.

(3)

(*d*) Find the range of fg.

(1)

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16.

6. The function f is defined by

$$f: x \mapsto \frac{3-2x}{x-5}, \quad x \in \mathbb{R}, \quad x \neq 5.$$

(*a*) Find $f^{-1}(x)$.

(3)

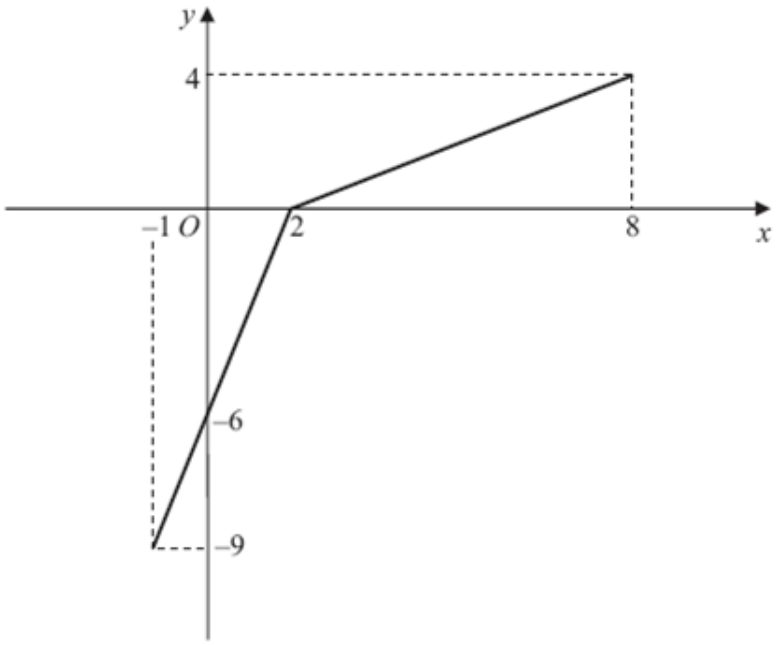

Figure 2

The function g has domain $-1 \leq x \leq 8$, and is linear from $(-1, -9)$ to $(2, 0)$ and from $(2, 0)$ to $(8, 4)$. Figure 2 shows a sketch of the graph of $y = g(x)$.

(*b*) Write down the range of g.

(1)

(*c*) Find gg(2).

(2)

(*d*) Find fg(8).

(2)

(*e*) On separate diagrams, sketch the graph with equation

(i) $y = |g(x)|$,

(ii) $y = g^{-1}(x)$.

Show on each sketch the coordinates of each point at which the graph meets or cuts the axes.

(4)

(*f*) State the domain of the inverse function g^{-1}.

(1)

17.

4. The function f is defined by

$$f : x \mapsto |2x - 5|, \quad x \in \mathbb{R}.$$

(*a*) Sketch the graph with equation $y = f(x)$, showing the coordinates of the points where the graph cuts or meets the axes.

(2)

(*b*) Solve $f(x) = 15 + x$.

(3)

The function g is defined by

$$g : x \mapsto x^2 - 4x + 1, \quad x \in \mathbb{R}, \quad 0 \leq x \leq 5.$$

(*c*) Find fg(2).

(2)

(*d*) Find the range of g.

(3)

18.

6.

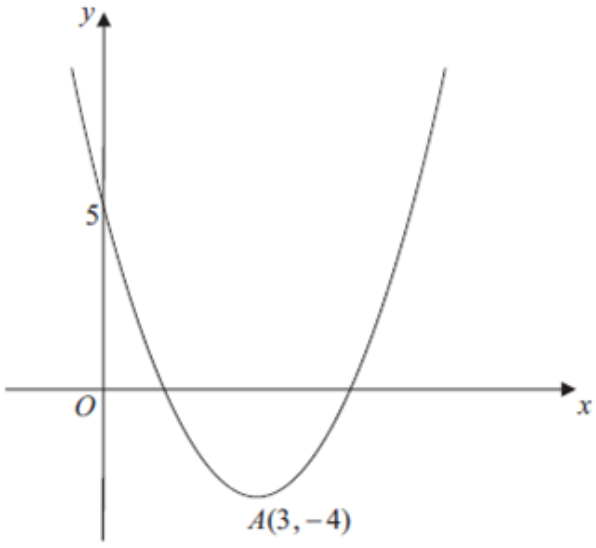

Figure 2

Figure 2 shows a sketch of the curve with the equation $y = f(x)$, $x \in \mathbb{R}$.

The curve has a turning point at $A(3, -4)$ and also passes through the point $(0, 5)$.

The curve has a turning point at $A(3, -4)$ and also passes through the point $(0, 5)$.

(a) Write down the coordinates of the point to which A is transformed on the curve with equation

(i) $y = |f(x)|$,

(ii) $y = 2f(\frac{1}{2}x)$.

(4)

(b) Sketch the curve with equation $y = f(|x|)$.

On your sketch show the coordinates of all turning points and the coordinates of the point at which the curve cuts the y-axis.

(3)

The curve with equation $y = f(x)$ is a translation of the curve with equation $y = x^2$.

(c) Find $f(x)$.

(2)

(d) Explain why the function f does not have an inverse.

(1)

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19.

5. Sketch the graph of $y = \ln |x|$, stating the coordinates of any points of intersection with the axes.

(3)

20.

6.

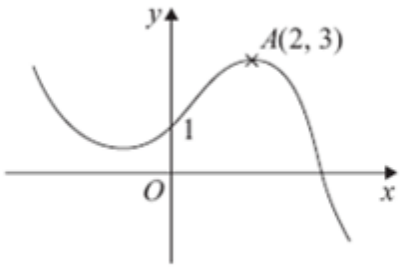

Figure 1

Figure 1 shows a sketch of the graph of $y = f(x)$.

The graph intersects the y-axis at the point $(0, 1)$ and the point $A(2, 3)$ is the maximum turning point.

Sketch, on separate axes, the graphs of

(i) $y = f(-x) + 1$,

(ii) $y = f(x + 2) + 3$,

(iii) $y = 2f(2x)$.

On each sketch, show the coordinates of the point at which your graph intersects the y-axis and the coordinates of the point to which A is transformed.

(9)

21.

9. (i) Find the exact solutions to the equations

(a) $\ln(3x - 7) = 5$,

(3)

(b) $3^x e^{7x+2} = 15$.

(5)

(ii) The functions f and g are defined by

$$f(x) = e^{2x} + 3, \qquad x \in \mathbb{R},$$

$$g(x) = \ln(x - 1), \qquad x \in \mathbb{R}, \quad x > 1.$$

(a) Find f^{-1} and state its domain.

(4)

(b) Find fg and state its range.

(3)